Journal of Fluids and Structures 18 (2003) 93–101

JOURNAL OF FLUIDS AND STRUCTURES

www.elsevier.nl/locate/jnlabr/yjfls

Effects of flexible walls on radiated sound from a turbulent boundary layer

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Received 9 October 2002; accepted 13 May 2003

Abstract

In this paper, radiated sound from low Mach number turbulent boundary layers is studied using Lighthill's analogy. The focus is on investigating the behavior of wall pressure fluctuations on a flexible wall boundary. Comparisons are made between the results from rigid and flexible walls. At low frequencies, the flexible wall results have shown much higher levels of power spectra. At high frequencies, the spectra of the flexible wall sound radiation merge with those of the rigid wall sound radiation. These trends are in agreement with other experimental and theoretical results in literature. This theoretical study of the two-dimensional turbulent boundary layer clarifies the effects of a flexible wall on radiated sound in low Mach number flows.

1. Introduction

Early in 1950s, Lighthill (1952, 1954) established the Lighthill analogy stress tensors as the sound source of turbulence. Curle (1955) investigated wall boundary effects using Lighthill's analogy and concluded that pressure fluctuations on the wall boundary generated radiative sound of dipole type. However, Powell (1964) stated that for turbulent boundary layers over smooth rigid wall surfaces, the Lighthill stress tensors behaved as quadrupoles. Other researchers have studied sound radiation from bursting and transitions in turbulent boundary layers (Landahl, 1975; Lauchle, 1981). Since the geometrical size within which bursting and transition phenomena occur is much smaller than the entire region of the turbulent boundary layer, their effective sound radiation may be neglected, according to measured results. Recent interest in automobile and aircraft cabin noise excited by turbulent boundary layers has also been explored (e.g., Maestrello, 1999; Wu et al., 1997).

There are two types of wall stresses that produce radiative noise sources, i.e., wall-pressure and shear-stress fluctuations. For low Mach number flows, Howe (1979) found that wall shear stresses were unlikely to be the dominant factor responsible for sound radiation. Therefore, fluctuation wall pressure has been identified as the dominant source of radiated sound from turbulent boundary layers. However, power spectra of radiated sound pressure from two-dimensional turbulent boundary layers over a rigid surface have been shown to be similar to that of the quadrupole sound radiation (e.g., Blake, 1986). This result supports the argument that the radiated sound from the turbulent boundary layer over a smooth, nonmoving, impervious rigid surface is quadrupole type. On the other hand, in a turbulent boundary layer over a flexible wall, the small displacement of the wall, excited by the turbulent fluctuation pressure, can possibly exacerbate the wall pressure fluctuation, and can thus generate net dipole-type sound radiation. The following discussion is to theoretically quantify such effects and to compare with the results from rigid wall cases.

It needs be pointed out that the weak coupling assumption (Graham, 1997) is accepted in the current study. This assumption corresponds to the assumption implicit in acoustic analogy analyses that the basic turbulence structure is

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0889-9746/$ - see front matter
doi:10.1016/S0889-9746(03)00081-1

essentially unaffected by the acoustic motions. The predictions that were obtained from the weak coupling approximation have been shown to work well in a wide range of cases (Graham, 1997), except for the differences between the theoretical results and measured data cited by Frendi (1997) in supersonic flow cases. For the current case of $M \ll 1$, the adoption of the decoupled approach is well justified.

2. Fluctuation wall pressure on a flexible wall

Considering low Mach number ($M \ll 1$), zero-pressure gradient flow of an isentropic Stokes compressible fluid, Lighthill (1952, 1954) showed that

$$\frac{1}{c_o^2}\frac{\partial^2 p}{\partial t^2} - \nabla^2 p = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}, \tag{1}$$

where

$$T_{ij} = \rho_o u_i u_j - e_{ij}, \quad e_{ij} = \mu_o\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\delta_{ij}\frac{\partial u_k}{\partial x_k}\right),$$

u_i, p are the fluctuation components of velocity and pressure, respectively. The ρ_o, c_o and μ_o are the mean flow density, sound speed and viscosity, respectively. These flow properties are assumed to be constant in this problem. In addition, the mean flow velocity is assumed to be $\boldsymbol{V}_o = V_o(y_2)\boldsymbol{i}$ and the convective effects in acoustic equations due to the mean velocity are neglected. The coordinate system is shown in Fig. 1.

Define a Green's function as

$$\frac{1}{c_o^2}\frac{\partial^2 G}{\partial \tau^2} - \nabla^2 G = \delta(x_1 - y_1, x_2 - y_2, t - \tau), \tag{2}$$

where $\boldsymbol{x}$ and $\boldsymbol{y}$ are the coordinates in the radiation sound region and the source region, respectively. Using a conjunct operation

$$\int [G*(Eq.(1)) - p*(Eq.(2))]\,\mathrm{d}y_1\,\mathrm{d}y_2\,\mathrm{d}\tau,$$

we obtain

$$p(x_1, x_2, t) = \int T_{ij}\frac{\partial^2 G}{\partial y_i \partial y_j}\,\mathrm{d}y_1\,\mathrm{d}y_2\,\mathrm{d}\tau + \int_{y_2=0}\left[p\frac{\partial G}{\partial y_2} - G\frac{\partial p}{\partial y_2}\right]\mathrm{d}y_1\,\mathrm{d}\tau. \tag{3}$$

The second term on the right-hand side (r.h.s.) of Eq. (3) is the surface integration along the boundary wall. Using the product theorem for Fourier transforms, it becomes

$$\int_{y_2=0}\left[p\frac{\partial G}{\partial y_2} - G\frac{\partial p}{\partial y_2}\right]\mathrm{d}y_1\,\mathrm{d}\tau = \frac{1}{(2\pi)^2}\int_{y_2=0}\left\{\hat{p}(k_1,\omega)\frac{\partial \hat{G}}{\partial y_2}(-k_1,-\omega) - \hat{G}(-k_1,-\omega)\frac{\partial \hat{p}}{\partial y_2}(k_1,\omega)\right\}\mathrm{d}k_1\,\mathrm{d}\omega, \tag{4}$$

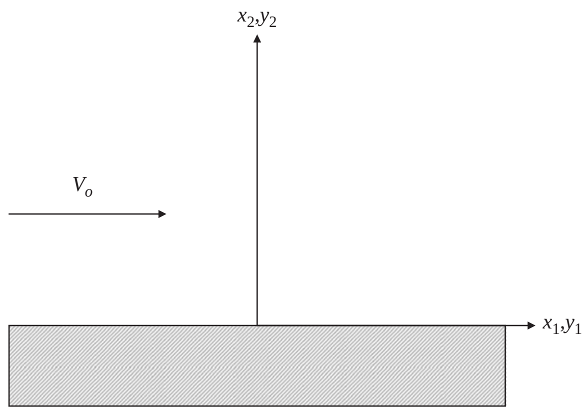

Fig. 1. The coordinate system used in this study.

where $\hat{f}$ is the Fourier transform of f from (y_1, y_2, τ) to (k_1, y_2, ω), defined as

$$\hat{f}(k_1, y_2, \omega) = \int_{-\infty}^{\infty} f(y_1, y_2, \tau) \mathrm{e}^{-\mathrm{i}(k_1 y_1 - \omega\tau)}\, \mathrm{d}y_1\, \mathrm{d}\tau. \tag{5}$$

Next, we try to express the surface fluctuation pressure and its derivative in Eq. (4) in terms of the surface displacement and the flexible wall dynamic property.

On the wall surface, neglecting viscous effects in the fluctuation part of the y_2-direction momentum, we have

$$-\frac{1}{\rho_o}\frac{\partial p}{\partial y_2} = \frac{\partial v_2}{\partial t} + v_1 \frac{\partial v_2}{\partial y_1} + v_2 \frac{\partial v_2}{\partial y_2}. \tag{6}$$

By assuming that v_1 and v_2 are small disturbances and therefore the higher order terms can be neglected, Eq. (6) becomes

$$\frac{\partial p}{\partial y_2} = -\rho_o \frac{\partial v_2}{\partial t}; \tag{7}$$

i.e.,

$$\frac{\partial \hat{p}}{\partial y_2}(k_1, \omega) = \rho_o \omega^2 \hat{\xi}(k_1, \omega), \tag{8}$$

where ξ is the vertical displacement of the flexible wall surface. In addition, on the flexible surface, we have

$$\hat{p}_w(k_1, \omega) = Z(k_1, \omega)\hat{\xi}(k_1, \omega), \tag{9}$$

where $Z(k_1, \omega)$ is the wall impedance based on displacement. Substitution of Eqs. (8) and (9) into Eq. (4) yields

$$\int_{y_2=0} \left[p\frac{\partial G}{\partial y_2} - G\frac{\partial p}{\partial y_2} \right] \mathrm{d}y_1\, \mathrm{d}\tau = \frac{1}{(2\pi)^2} \int_{y_2=0} \hat{\xi}(k_1, \omega) \left[Z(k_1, \omega) \frac{\partial \hat{G}}{\partial y_2}(-k_1, -\omega) - \rho_o \omega^2 \hat{G}(-k_1, -\omega) \right] \mathrm{d}k_1\, \mathrm{d}\omega. \tag{10}$$

If we let the Green's function G on the wall surface satisfy

$$\frac{\partial \hat{G}}{\partial y_2}(k_1, \omega) = \frac{\rho_o \omega^2 \hat{G}(k_1, \omega)}{Z(-k_1, -\omega)}, \tag{11}$$

the second term on the r.h.s. of Eq. (3) becomes zero. Hence,

$$p(x_1, x_2, t) = \int T_{ij} \frac{\partial^2 G}{\partial y_i \partial y_j} \mathrm{d}y_1\, \mathrm{d}y_2\, \mathrm{d}\tau. \tag{12}$$

Eq. (12) shows that the fluctuation pressure is determined by the Lighthill stress T_{ij} and the Green's G. The effects of a flexible wall are represented by the expression of G. It should be noted that because of the weak coupling assumption as stated previously, the Lighthill stress, T_{ij}, behaves the same as on a rigid wall.

On the wall boundary, let $x_2 = 0$ in Eq. (12), and we have

$$p_w(x_1, t) = p(x_1, 0, t) = \int T_{ij} \frac{\partial^2 G_{x_2=0}}{\partial y_i \partial y_j} \mathrm{d}y_1\, \mathrm{d}y_2\, \mathrm{d}\tau. \tag{13}$$

Green's function that satisfies the boundary condition expressed in Eq. (11) can be obtained by using Dowling's (1983) results, with the special case of $\gamma_0 = \gamma_1 = \gamma$, $U_1 = 0$ and $h = 0$ in her results. Note that in Dowling's case, the conditions are for inward waves, while in this case the sound waves are outward waves. The expression of Green's function is then

$$G_{x_2=0} = \frac{1}{(2\pi)^2} \int F(k_1, \omega) \mathrm{e}^{\mathrm{i}g}\, \mathrm{d}k_1\, \mathrm{d}\omega, \tag{14}$$

where

$$F(k_1, \omega) = \frac{Z(-k_1, -\omega)}{\rho_o \omega^2 + \mathrm{i}\gamma Z(-k_1, -\omega)},$$

$$g = k_1(y_1 - x_1) - \omega(\tau - t) - \gamma y_2,$$

and

$$\gamma^2 = \frac{\omega^2}{c_o^2} - k_1^2.$$

For radiated sound, only $|k_1| < \omega/c_o$ is nondecaying; therefore γ is a positive real number (when $\omega > 0$). Substitute Eq. (14) into Eq. (13) and switch the order of the multi-integrations to get

$$p_w(x_1, t) = \frac{1}{(2\pi)^2} \int F(k_1, \omega)[-(k_1\delta_{i1} - \gamma\delta_{i2})(k_1\delta_{j1} - \gamma\delta_{j2})]\hat{T}_{ij}(-k_1, y_2, -\omega)\mathrm{e}^{-\mathrm{i}\gamma y_2}\mathrm{e}^{-\mathrm{i}(k_1x_1-\omega t)}\,\mathrm{d}y_2\,\mathrm{d}k_1\,\mathrm{d}\omega. \tag{15}$$

By replacing $-k_1$ and $-\omega$ with k_1 and ω, respectively, it can be deduced from Eq. (15) that

$$\hat{p_w}(k_1, \omega) = -\int F(-k_1, -\omega)(k_1\delta_{i1} - \gamma\delta_{i2})(k_1\delta_{j1} - \gamma\delta_{j2})\mathrm{e}^{\mathrm{i}\gamma y_2}\hat{T}_{ij}\,\mathrm{d}y_2. \tag{16}$$

It can be seen that the difference between a rigid wall and a flexible wall is reflected in F. On a rigid wall, $Z \to \infty$, we have

$$F(-k_1, -\omega) = -\frac{1}{\mathrm{i}\gamma}, \tag{17}$$

and Eq. (16) becomes

$$\hat{p}_{w\mathrm{rigid}}(k_1, \omega) = -\frac{\mathrm{i}}{\gamma}\int (k_1\delta_{i1} - \gamma\delta_{i2})(k_1\delta_{j1} - \gamma\delta_{j2})\mathrm{e}^{\mathrm{i}\gamma y_2}\hat{T}_{ij}\,\mathrm{d}y_2. \tag{18}$$

This expression is the same as the result in Howe (1979), although the boundary conditions for the Green's function in that paper were for the rigid wall surface only. On a flexible wall,

$$F(-k_1, -\omega) = \frac{Z(k_1, \omega)}{\rho_o\omega^2 - \mathrm{i}\gamma Z(k_1, \omega)}, \tag{19}$$

where the displacement impedance for a flexible wall, $Z(k_1, \omega)$, is yet to be determined.

For a two-dimensional plate, the equation for bending vibration is (Ffowcs-Williams, 1965)

$$m\frac{\partial^2 \xi}{\partial t^2} + B\frac{\partial^4 \xi}{\partial x^4} = -(p_w - p_a), \tag{20}$$

where m is the mass per unit area of the plate, p_a is the acoustic pressure fluctuation due to plate motion, and B is the bending stiffness. Both m and B are the material properties of the plate. The pressure force, $p_w - p_a$, is called "blocked" boundary pressure by Graham (1997). In the transformed domain, Eq. (20) becomes

$$-m\omega^2\hat{\xi} + Bk_1^4\hat{\xi} = -(\hat{p}_w - \hat{p}_a). \tag{21}$$

The induced acoustic pressure fluctuation can be expressed as (e.g., Ross, 1976; Howe, 1998)

$$\hat{p}_a = -\frac{\mathrm{i}}{\gamma}\rho_o\omega^2\hat{\xi}. \tag{22}$$

Hence, from Eq. (21),

$$Z(k_1, \omega) = m\omega^2 - \frac{\mathrm{i}}{\gamma}\rho_o\omega^2 - Bk_1^4. \tag{23}$$

Substitute this expression into Eq. (19) to get

$$F(-k_1, -\omega) = \frac{m\omega^2 - Bk_1^4 - \mathrm{i}\rho_o\omega^2/\gamma}{-\mathrm{i}\gamma(m\omega^2 - Bk_1^4)}. \tag{24}$$

For rigid walls, $B \to \infty$, and Eq. (24) reduces to Eq. (17).

If we compare the fluctuation wall pressure on flexible and rigid walls, we have

$$\frac{\hat{p}_{w\mathrm{flex}}}{\hat{p}_{w\mathrm{rigid}}} = \frac{F_{\mathrm{flex}}(-k_1, -\omega)}{F_{\mathrm{rigid}}(-k_1, -\omega)} = 1 - \mathrm{i}\frac{\rho_o\omega^2}{\gamma(m\omega^2 - Bk_1^4)}. \tag{25}$$

There are singularities at the sonic wavenumber $|k_1| = \omega/c_o$ and at $|k_1| = (m\omega^2/B)^{1/4}$. These singularities may not exist if viscous effects are present. What is important is that Eq. (25) shows

$$\left|\frac{\hat{p}_{w\mathrm{flex}}}{\hat{p}_{w\mathrm{rigid}}}\right| > 1$$

for any frequency and wavenumber. That means the magnitude of wall fluctuation pressure on a flexible wall is always greater than that on a rigid wall. This will cause higher radiated noise from a flexible wall, as shown in the next section. The excessive amount of pressure fluctuation on a flexible wall is determined by the imaginary part in Eq. (25). For small k_1, this part approximately becomes

$$\beta = \frac{\rho_o c_o}{m\omega},$$

where β is called the fluid loading factor in Blake (1986). It can be seen that: (a) the excessive amount of fluctuation pressure is proportional to the characteristic acoustic impedance, $\rho_o c_o$, of the fluid; (b) at lower frequencies the secondary sound radiation due to flow-excited surface vibration on a flexible wall is more significant, the fact compatible with results shown in Blake (1986). The characteristic impedances for air and water are approximately 415 and 1.48×10^6 mksRayls, respectively. If we consider a steel plate with thickness of 1 cm and $\rho_{\text{plate}} = 8 \times 10^3\ \text{kg/m}^3$, we have

$$\frac{\rho_o c_o}{\rho_{\text{plate}} h\omega} = \frac{5.19}{\omega(1/\text{s})}$$

in air, and

$$\frac{\rho_o c_o}{\rho_{\text{plate}} h\omega} = \frac{1.85 \times 10^4}{\omega(1/\text{s})}$$

in water. Therefore, in the range of radiated sound frequency of 10 Hz–100 kHz, the secondary sound radiation in air-flow is not significant. That is why in most of the air-flow/structure coupling problems, the secondary sound radiation can be neglected, unless in low frequency ranges. However, in hydrodynamic problems, which Blake (1986) called heavy fluid problems, the secondary radiation is dominant, the fact that is shown quantitatively in the following section. This secondary radiation was called flow-excited noise by Vecchio and Wiley (1973), in contrast to flow-induced noise which is due to the direct radiation from the wall turbulence.

3. Sound radiation from a flexible wall

If the convection effects from the turbulent boundary layer can be neglected, the radiated sound pressure can be related with the surface pressure as shown by Tam (1975):

$$\hat{p}(k_1, x_2, \omega) = \hat{p}_w(k_1, \omega)\mathrm{e}^{\mathrm{i}\gamma x_2}. \tag{26}$$

Therefore,

$$\begin{aligned} p(x_1, x_2, t) &= \frac{1}{(2\pi)^2} \int \hat{p}_w(k_1, \omega)\mathrm{e}^{\mathrm{i}(\gamma x_2 + k_1 x_1 - \omega t)}\, \mathrm{d}k_1\, \mathrm{d}\omega \\ &= \frac{1}{(2\pi)^2} \int (-\mathrm{i}\gamma) p_{w\text{rigid}}(y_1', t') F(-k_1, -\omega)\mathrm{e}^{\mathrm{i}\gamma x_2}\mathrm{e}^{\mathrm{i}[k_1(x_1 - y_1') - \omega(t - t')]}\, \mathrm{d}y_1'\, \mathrm{d}t'\, \mathrm{d}k_1\, \mathrm{d}\omega. \end{aligned} \tag{27}$$

The power spectrum of radiated sound is defined as

$$P(\omega) = \int \langle p(x_1, x_2, t)p(x_1, x_2, t+\tau)\rangle \mathrm{e}^{\mathrm{i}\omega\tau}\, \mathrm{d}\tau, \tag{28}$$

where "$\langle\ \rangle$" denotes ensemble averages. From Eq. (26) and assuming p_w to be a stationary random function, Tam (1975) showed that

$$P(\omega) = \frac{1}{(2\pi)^4} \int_{|k_1| < \omega/c_o} \hat{R}(k_1, \omega)\, \mathrm{d}k_1, \tag{29}$$

where

$$\hat{R}(k_1, \omega) = \int R(\xi, \eta)\mathrm{e}^{-\mathrm{i}(k_1\xi - \omega\eta)}\, \mathrm{d}\xi\, \mathrm{d}\eta \tag{30}$$

and

$$R(\xi, \eta) = \langle p_w(y_1', t')p_w(y_1'', t'')\rangle, \tag{31}$$

in which $\xi = y_1' - y_1''$ and $\eta = t' - t''$. By substituting Eq. (27) into Eq. (28), it can be proved that

$$P(\omega) = \frac{1}{(2\pi)^4} \int_{|k_1|<\omega/c_o} \hat{R}_{\text{rigid}}(k_1,\omega)\gamma^2 F(-k_1,-\omega)F(k_1,\omega)\,\mathrm{d}k_1. \tag{32}$$

Comparing the above expression with Eq. (29), we have

$$\hat{R}(k_1,\omega) = \hat{R}_{\text{rigid}}(k_1,\omega)\gamma^2 F(-k_1,-\omega)F(k_1,\omega). \tag{33}$$

Therefore, substituting Eq. (24) into the above expression, we have

$$\hat{R}(k_1,\omega) = \hat{R}_{\text{rigid}}(k_1,\omega) + \frac{\rho^2\omega^4}{\gamma^2(m\omega^2 - Bk_1^4)^2}\hat{R}_{\text{rigid}}. \tag{34}$$

Eqs. (32) and (34) yield

$$P(\omega) = P_{\text{rigid}}(\omega) + \frac{1}{(2\pi)^4}\int_{|k_1|<\omega/c_o} \frac{\rho^2\omega^4}{\gamma^2(m\omega^2 - Bk_1^4)^2}\hat{R}_{\text{rigid}}\,\mathrm{d}k_1. \tag{35}$$

Eq. (35) shows that there is an excessive amount of the radiated sound spectrum of a flexible wall over that of a rigid wall.

In order to quantitatively estimate the r.h.s. of Eq. (35), $\hat{R}_{\text{rigid}}$ has to be known. The wall pressure cross-correlations beneath turbulent boundary layers have been studied during the last 40 years or so (e.g., Bull, 1996). In this paper, an experimental result by Maestrello (1967) for a rigid wall surface has been selected to derive analytical expressions for the power spectra. Maestrello's three-Gaussian-correlation model can lead to analytical expressions of power spectra as shown later in this paper and also in Tam's (1975). It is important that the behavior and comparison of the solutions be analyzed based on the closed-form solutions in this case, in order to recognize and exclude singularity effects in the solutions to be discussed later. There was a substantial amount of experimental data in Maestrello's paper (1967) to support the model, and the model could be applied to relatively high subsonic Mach numbers. Although strictly speaking the results presented in this paper is applicable to $M \ll 1$, they could be stretched to a higher Mach number range as long as the convective effects in acoustic equations could be neglected.

Assuming uniformity in the span-wise direction in this case, Maestrello's correlation can be expressed as

$$R(\xi,\eta) = \bar{\tau}_w^2 \exp\left(-\frac{|\xi|}{U_c\theta}\right)\left\{\sum_{i=1}^{3}\frac{A_i\alpha_i}{\alpha_i^2 + (\frac{1}{FU})^2(\xi - U_c\eta)^2}\right\}\Big/\sum_{i=1}^{3}\frac{A_i}{\alpha_i}, \tag{36}$$

where $\bar{\tau}_w$ is the mean flow wall shear stress, A_i and α_i are the constants defined in Maestrello (1967), U is the freestream velocity, U_c is the convective velocity of eddies in the turbulent boundary layer and is chosen as $0.8U$, $F = \delta^*/U$, $U_c\theta/\delta^* = 17.0$, in which δ^* is the displacement thickness of the turbulent boundary layer. Hence, after the Fourier transform we can have

$$\hat{R}_{\text{rigid}}(k_1,\omega) = \frac{2\pi\bar{\tau}_w^2}{(U_c^2\theta/HU)\sum_{i=1}^{3}(A_i/\alpha_i)}\frac{1}{(\omega/U_c - k_1)^2 + 1/U_c^2\theta^2}\sum_{i=1}^{3} A_i \exp\left(-\alpha_i\frac{HU\omega}{U_c}\right). \tag{37}$$

Substitution of Eq. (37) into Eq. (35) gives, in a dimensionless format,

$$\begin{aligned} LP = {}& \frac{P(\omega)(2\pi)^3}{H\bar{\tau}_w^2} = \frac{P_{\text{rigid}}(\omega)(2\pi)^3}{H\bar{\tau}_w^2} \\ &+ \frac{U/U_c}{\sum_{i=1}^{3}A_i\alpha_i}\frac{(\rho_o/\rho_{\text{plate}})^2(\delta^*/h)^2}{U_c\theta/\delta^*}\frac{\sum_{i=1}^{3}A_i\exp(-\alpha_i QU/U_c)}{QM} \\ &\times \int_{-1}^{1}\frac{1}{(1-y^2)[1 - Q^2M^2y^4(h/\delta^*)^2E/12(1-\sigma^2)\rho_{\text{plate}}c_o^2]^2} \\ &\times \frac{\mathrm{d}y}{Q^2(U/U_c - My)^2 + (U_c\theta/\delta^*)^{-2}}, \end{aligned} \tag{38}$$

where $y = k_1c_o/\omega$, $Q = \delta^*\omega/U$, $M = U/c_o$, and m and B have been replaced by $\rho_{\text{plate}}h$ and $Eh^3/[12(1-\sigma^2)]$, where E is the elasticity of the plate and σ is the plate Poisson ratio (Timoshenko and Woinowsky-Krieger, 1959). The rigid power spectrum can be obtained as

$$LP_{\text{rigid}} = \frac{P_{\text{rigid}}(\omega)(2\pi)^3}{H\bar{\tau}_w^2} = \frac{1}{(U_c/U)\sum_{i=1}^{3} A_i\alpha_i}\left[\tan^{-1}\frac{2Q(U_c\theta/\delta^*)M}{1+Q^2(U_c\theta/\delta^*)^2(U^2/U_c^2-M^2)}\right]\sum_{i=1}^{3} A_i \exp\left(-\alpha_i Q\frac{U}{U_c}\right). \quad (39)$$

Fig. 2 is a sample calculation for comparison of the logarithmic LP at $M = 0.1$ on rigid and flexible walls, with Q ranging from 10^{-4} to 10, using the values for the constants given in Maestrello (1967). The physical properties are based on a steel plate as the wall and water as the fluid, with $E = 2 \times 10^{11}$ N/m^2, $\rho_{\text{plate}} = 8 \times 10^3$ kg/m^3, $\sigma = 0.3$, $h = 10^{-2}$ m, $h/\delta^* = 1.0$, $\rho_o = 1.0 \times 10^3$ kg/m^3 and $c_o = 1.48 \times 10^3$ m/s. The integration range in Eq. (38) is from -0.9 to 0.9, to

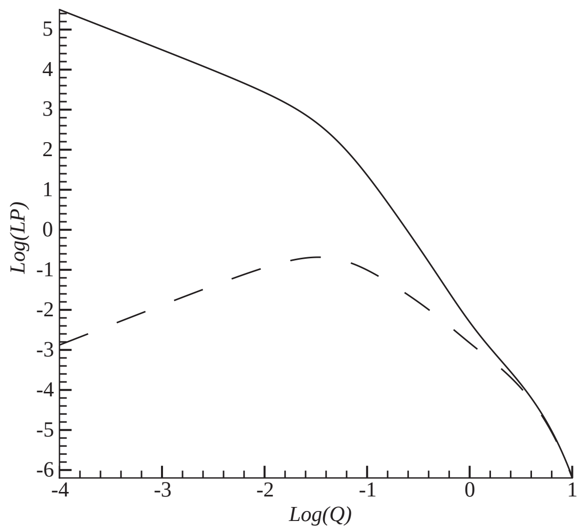

Fig. 2. Comparison of radiated sound spectra of the flexible and rigid walls. The solid curve is for flexible walls and the dashed curve is for rigid walls.

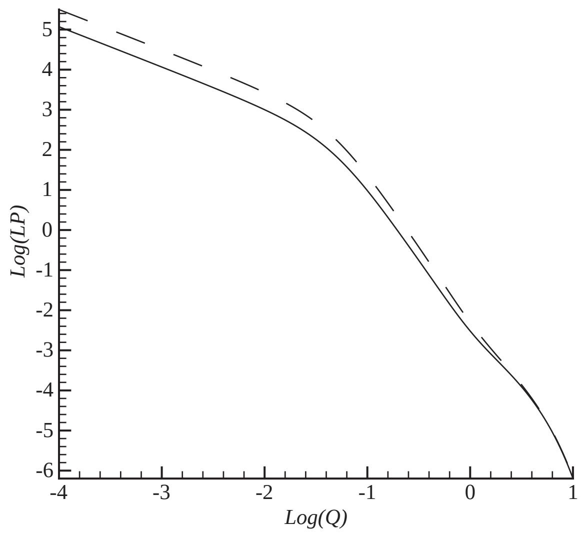

Fig. 3. Effects of integration ranges on radiated sound spectra of the flexible walls, with different integration ranges: the solid curve is with the range from -0.5 to 0.5; the dashed curve is with the range from -0.9 to 0.9.

avoid singularities at the sonic wave number. The bending wave number singularity at $|k_1| = (m\omega^2/B)^{1/4}$ does not occur with the current physical property values for low Mach number flow. The integration uses the Romberg integration algorithm in Press et al. (1986). It can be seen that at low frequencies, the flexible wall has much higher radiated sound than the rigid wall. At high frequencies, the two spectra merge together. Such phenomena can also be discerned from the expression on the r.h.s. of Eq. (38), since the difference (the second term on the r.h.s. of Eq. (38)) between the flexible and rigid wall is proportional to $1/Q$ at small Q values. These trends agree with several results for panel radiation from turbulent flow-excited wall vibration presented in Blake (1986).

In order to clarify the concern that the excessive radiation might be due to the singularity at sonic wavenumber in the integration, a substantially smaller integration range for y from -0.5 to 0.5 has been tested for the calculation. Fig. 3 is the comparison of the flexible wall spectra using the smaller and larger ranges of wavenumber integrations for calculation. It can be seen that only a slight difference is shown. That means that the excessive amount of sound radiation from a flexible wall is not caused by the singularity at the sonic wavenumber. Bergeron (1974) analyzed this singularity in greater detail and showed that this nonintegrable singularity arose because the two-dimensional turbulent source region was considered to be infinite extent, and the sound field from each source element did not decrease rapidly enough with distance for the integrated effect to be finite. He demonstrated that when the source region had finite extent there was still a singularity for spectral elements with sonic phase speeds but the singularity was integrable. Howe (1979) found that when viscosity was included the pressure spectrum remained finite.

4. Conclusion

Flexible wall effects have been represented by a Green's function which satisfies the specified boundary condition related with the flexible wall impedance. The wall pressure fluctuation can thus be compared with different flexible wall properties. The study has shown that the excessive wall pressure fluctuation on a flexible wall is related with the fluid loading factor, which is determined by the characteristic impedance of the fluid. A mathematical model of the rigid wall cross-correlation function of fluctuation wall pressure is then modified to include flexible wall effects. These relationships have been employed to calculate the power spectra of the radiated sound field from flexible walls. Comparisons between the power spectra of flexible and rigid wall radiation have shown that, at low frequencies, the flexible wall has much higher levels of spectra. At high frequencies, the spectra of the flexible wall merge with those of the rigid wall. These trends are in agreement with other experimental and theoretical results in literature. The effects of singularity at the sonic wavenumber have been discussed and their influence on the excessive amount of radiation from a flexible wall is excluded.

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